BEHAVIOUR OF SPIN-HALF PARTICLES IN CURVED SPACE-TIME

BANIBRATA MUKHOPADHYAY
S.N.Bose National Centre for Basic Sciences, Block-JD, Sector-III, Salt Lake, Calcutta-700098, India
E-mail: bm@boson.bose.res.in *

We study the behaviour of spin-half particles in curved space-time. Since Dirac equation gives the dynamics of spin-half particles, we mainly study the Dirac equation in Schwarzschild, Kerr, Reissner-Nordström geometry. Due to the consideration of existence of black hole in space-time (the curved space-time), particles are influenced and equation will be modified. As a result the solution will be changed from that due to flat space.

Here we study the interaction of particles having spin half with a black hole. We are familiar with Dirac equation in flat space by which we can investigate the behaviour of spin-half particle. After writing the Dirac equation in curved space-time particularly in Kerr or Kerr-Newman geometry using Newman-Penrose formalism[1] general relativistic effect was introduced and the equation was generalised and modified. As the space-time is asymptotically flat, far away from the black hole the modified Dirac equation in Kerr-Newman, Reissner-Nordström or Schwarzschild geometry and the interaction of particles with space-time reduces into that of the flat space.

One of the most important solutions of Einstein's equation is that of the space-time around and inside an isolated black hole. The spacetime at a large distance is flat and Minkowskian where usual quantum mechanics is applicable, while the spacetime closer to the singularity is so curved that no satisfactory quantum field theory could be developed as yet. An intermediate situation arises when a weak perturbation (due to, say, gravitational, electromagnetic or Dirac waves) originating from infinity impinges on a black hole, interacting with it. The resulting wave is partially transmitted into the black hole through the horizon and partially scatters off to infinity. Particularly interesting is the fact that whereas gravitational and electromagnetic radiations were found to be amplified in some range of incoming frequencies, Chandrasekhar[1] predicted that no such amplifications should take place for Dirac waves because of the very nature of the potential experienced by the incoming fields. However, these later conclusions were drawn by Chandrasekhar using asymptotic solutions. Here we revisit this important problem in a spatially complete manner to study the nature of the Dirac spinors as a function of the Kerr parameter, rest mass and frequency of incoming particle.

The separated Dirac equation in Kerr geometry (into radial and angular part) is given by Chandrasekhar[1]. Following Mukhopadhyay & Chakrabarti[2] we can solve the equation in generalised Kerr-Newman geometry where the basis vectors of the equation should be expressed in terms of different coefficients of corresponding metric given by

*Presently working in: Physical Research Laboratory, Navrangpura, Ahmedabad-380009, India E-mail: bm@prl.ernet.in

$$g^{ij} = \begin{pmatrix} \Sigma^2/\rho^2\Delta & 0 & 0 & 2aMr/\rho^2\Delta \\ 0 & -\Delta/\rho^2 & 0 & 0 \\ 0 & 0 & -1/\rho^2 & 0 \\ 2aMr/\rho^2\Delta & 0 & 0 & -(\Delta - a^2 sin^2\theta)/\rho^2\Delta sin^2\theta \end{pmatrix} \tag{1}$$

where the different symbols have their usual meaning[1,3]. For simplicity we can choose the angular momentum of the black hole to zero and space-time reduces to spherically symmetric Reissner-Nordström geometry. Following Mukhopadhyay & Chakrabarti[2] and Mukhopadhyay[3] we can reduce the radial Dirac equation into Schrödinger like equation by suitably transforming the independent variable r to $\hat{r}_*$ (which are related to each other logarithmically[2,3,4]) and defining new spinor fields as a certain linear combination of up and down spinors. Finally we solve the reduced radial equation by *IWKB* approximation method[2,3] and get reflection and transmission coefficients of Dirac particles as a function of rest mass and frequency of the incoming Dirac particle. The angular equation was already solved earlier[2]. Thus, combining the radial and angular solutions spatially complete solution of Dirac equation is obtained.

If we consider the incoming Dirac particle as charged one (like electron, positron etc.) then the covariance is appearing into the derivative of the equation not only due to the curvature of the space (affine connections Γ) but also due to the electromagnetic interaction[4]. Thus the covariant derivative reduces as

$$D_\mu = \partial_\mu + iq_1 A_\mu + q_2 \Gamma_\mu \tag{2}$$

where, q_1 and q_2 are coupling constants. A_μ is the electromagnetic gauge field.

Another important issue in this perspective is the super-radiance. In case of spherically symmetric Schwarzschild geometry there is no question of super-radiance[2] but for the case of Kerr or Kerr-Newman geometry there is a parameter region for which gravitational potential diverges. But it is seen that potential varies as $\frac{1}{(r-|\alpha|)^3}$ (α is a constant) close to the singular point. As because potential has a strongly attractive branch practically no particle can come out from its field as a result super-radiation of Dirac particle is impossible. Similar feature is seen in case of Reissner-Nordström geometry.

The reflection and transmission coefficients of the Dirac particles were found to distinguish strongly the solutions of different rest masses and different energies. The waves scattered off are distinctly different in different parameter regions. In a way, therefore, black holes can act as a mass spectrograph! Another interesting application of our method and solution would be to study the interactions of Hawking radiations in regions just outside the horizon.

References

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